All solutions to graded problems must be typed in LaTeX. All other problems can be typed or written, but must be completed. Graded problems must be organized in numerical order. All problem sets (both graded and non-graded) must be uploaded to Canvas as a single PDF document. Students can find the grading scale in our Syllabus.

Graded

Exercise 1. Prove the following:

Theorem. Let X be a topological space with topology $\mathscr{T}$ and let A be a subset of X. The set A is open in X if and only if $\mathscr{T}_A \subseteq \mathscr{T}$.

Exercise 2. Let $I = [0, 1]$. Compare (i.e. finer/coarser) the product topology on $I \times I$, the dictionary order on $I \times I$, and the topology $I \times I$ inherits as a subspace of $\mathbb{R} \times \mathbb{R}$ in the dictionary order topology.

Exercise 3. Show that the dictionary order topology on $\mathbb{R} \times \mathbb{R}$ is the same as the product topology $\mathbb{R}_d \times \mathbb{R}$, where $\mathbb{R}_d$ denotes $\mathbb{R}$ with the discrete topology. Compare this topology with the standard topology on $\mathbb{R}^2$.

Non-Graded

Exercise 4. A map $f : X \to Y$ is said to be an *open map* if for every open set U of X, the set $f(U)$ is open in Y. Show that $\pi_1 : X \times Y \to X$ and $\pi_2 : X \times Y \to Y$ are open maps.

Exercise 5. If $\mathscr{T}$ and $\mathscr{T}'$ are topologies on X and $\mathscr{T} \subset \mathscr{T}'$, what can be said about the corresponding subspace topologies on a subset Y of X? Prove your claim.